Review Problem 12

- Convert the following circuit to NAND/NOR form
Optimization via K-Maps to 2-level forms

- Readings: 2.11-2.12.2, 2.14
- Sum of Products form: the OR of several AND gates, inversions over only inputs
  - $F = \overline{X} + Y\overline{Z} + XYZ$
- Circuit diagram & inversions:
### On Sets and Off Sets

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>H</th>
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</thead>
<tbody>
<tr>
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- **On Set** is the set of input patterns where the function is TRUE.  
  \( \overline{x} \overline{y} \overline{z}, \overline{x}yz, \overline{x} \overline{y} \overline{z}, \overline{x} \overline{z} \)

- **Off Set** is the set of input patterns where the function is FALSE.  
  \( x \overline{y} \overline{z}, x \overline{y} z, xy \overline{z}, xy \overline{z} \)
# Two-Level Simplification

**Key Tool: The Uniting Theorem** — \( A (\overline{B} + B) = A \)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>F</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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</table>

\[ F = A \overline{B} + A B = A (\overline{B} + B) = A \]

- B's values change within the on-set rows
- \( B \) is eliminated, \( A \) remains

- A's values don't change within the on-set rows

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>G</th>
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<tbody>
<tr>
<td>0</td>
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</table>

\[ G = \overline{A} \overline{B} + A \overline{B} = (\overline{A} + A) \overline{B} = \overline{B} \]

- B's values stay the same within the on-set rows
- \( A \) is eliminated, \( B \) remains

- A's values change within the on-set rows

**Essence of Simplification:**

find two element subsets of the ON-set where only one variable changes its value. This single varying variable *can be eliminated!*
Karnaugh Maps

Karnaugh Map Method

K-map is an alternative method of representing the truth table that helps visualize adjacencies in up to 4 dimensions.

Beyond that, computer-based methods are needed.

2-variable K-map

3-variable K-map

4-variable K-map
Truth Tables to K-Maps

A B | F
---|---
0 0 | 0
0 1 | 0
1 0 | 1
1 1 | 0

A B | F
---|---
A A
B B

A B C D | H
---|---
0 0 0 0 | 0
0 0 0 1 | 0
0 0 1 0 | 0
0 0 1 1 | 1
0 1 0 0 | 0
0 1 0 1 | 0
0 1 1 0 | 0
0 1 1 1 | 1
1 0 0 0 | 1
1 0 0 1 | 1
1 0 1 0 | 0
1 0 1 1 | 1
1 1 0 0 | 0
1 1 0 1 | 0
1 1 1 0 | 1
1 1 1 1 | 1

A B
C D
A
B
C
D

C Suppressed
K-Map Simplification

K-Map Method Examples

F = A

G = \overline{B}

\text{Cout} = AB + B\text{Cin} + AC\text{Cin}

F(A,B,C) = A
K-Map Simplification (cont.)

More K-Map Method Examples, 3 Variables

\[
F(A,B,C) = \overline{A} \overline{B} \overline{C} + AB \overline{C} + A \overline{B} \overline{C} + AB C
\]

\[
F = AC + \overline{B} \overline{C}
\]

In the K-map, adjacency wraps from left to right and from top to bottom

\[F\] simply replace 1's with 0's and vice versa

\[
\overline{F}(A,B,C) = \overline{A} \overline{B} \overline{C} + \overline{A} B \overline{C} + \overline{A} B \overline{C} + A B \overline{C}
\]

\[
\overline{F} = BC + AC
\]
4-Variable K-Map

K-map Method Examples: 4 variables

\[ F = \overline{AD} + BD + \overline{BC} + A\overline{BD} \]

\[ F = D + \overline{BC} \]
K-Map Example

\[ F = (A \text{xor} C) \ast D + A\overline{C}D + \overline{A}B\overline{C}\overline{D} \]

\[ = A\overline{C}D + \overline{A}CD + A\overline{C}D + \overline{A}B\overline{C}D \]

\[ F = A\overline{C} + \overline{A}CD + \overline{ABC} \]