Review Problem 37

- If all gates have a delay of 1 ns, how long does a 4-bit adder take to compute?

\[ \text{A3 B3} \quad \text{A2 B2} \quad \text{A1 B1} \quad \text{A0 B0} \]

\[ \text{S3 C3 S2 C2 S1 C1 S0} \]

\[ 7 \text{ns} \]
Negative Numbers

- Need an efficient way to represent negative numbers in binary
  - Both positive & negative numbers will be strings of bits
  - Use fixed-width formats (4-bit, 16-bit, etc.)
- Must provide efficient mathematical operations
  - Addition & subtraction with potentially mixed signs
  - Negation (multiply by -1)

\[
\begin{align*}
S & \quad V_3 V_2 V_1 V_0 \\
0 & \quad \text{positive} \\
1 & \quad \text{negative}
\end{align*}
\]
Sign/Magnitude Representation

High order bit is sign: 0 = positive (or zero), 1 = negative

Three low order bits is the magnitude: 0 (000) thru 7 (111)

Number range for n bits = +/-2^{n-1} -1

Representations for 0:
Sign/Magnitude Addition

Signs same: add the magnitudes, keep the sign.

\[
\begin{align*}
0 & 0 & 1 & 0 & \text{(+2)} \\
+ & 0 & 1 & 0 & 0 & \text{(+4)} \\
\hline
& 1 & 1 & 1 & 0 & \# \\
\end{align*}
\]

Signs different: subtract smaller mag from the larger, keep the sign of the larger.

\[
\begin{align*}
0 & 0 & 1 & 0 & \text{(+2)} \\
+ & 1 & 1 & 0 & 0 & \text{(-4)} \\
\hline
5 & 1 & 0 & 0 & \text{(-6)} \\
\end{align*}
\]

Bottom line: Basic mathematics are too complex in Sign/Magnitude.
Idea: Pick negatives so that addition works

- Let $-1 = 0 - (+1)$:
  \[
  \begin{array}{c}
  0 \ 0 \ 0 \ 0 \ (0) \\
  - 0 \ 0 \ 0 \ 1 \ (+1) \\
  \hline
  1 \ 1 \ 1 \ 1 \ (-1)
  \end{array}
  \]

- Does addition work?
  \[
  \begin{array}{c}
  \times \ 1 \ 1 \ 1 \ 0 \\
  \hline
  0 \ 0 \ 1 \ 0 \ (+2) \\
  + 1 \ 1 \ 1 \ 1 \ (-1) \\
  \hline
  0 \ \ 0 \ \ 0 \ \ 1
  \end{array}
  \]

- Result: Two’s Complement Numbers
Two’s Complement

- Only one representation for 0: 0000
- One more negative number than positive number
- Fixed width format for both pos. & neg. numbers
Negating in Two’s Complement

- Flip bits & Add 1
- Negate \((0010)_2\) (+2)
  
  \[ \text{what is } -2 = -(0010) \]
  
  \[ = (1101 + 1) \]
  
  \[ = 1110 \]

- Negate \((1110)_2\) (-2)
  
  \[ \text{what is } -(1110) ? \]
  
  \[ = (0001 + 1) = 0010 \]
  
  \[ + 2 \]
Addition in Two’s Complement

\[
\begin{align*}
\text{0000} & \quad \text{1000} \\
\text{0010} \quad (+2) & \quad \text{1110} \quad (-2) \\
\text{0100} \quad (+4) & \quad \text{1100} \quad (-4) \\
\text{0110} & \quad \text{1010} \\
\text{0000} & \quad \text{1110} \\
\text{0010} \quad (+2) & \quad \text{1100} \quad (-4) \\
\text{1110} & \quad \text{1010} \\
\end{align*}
\]

\[
\begin{align*}
= -1110 & = -(-1110) = -(0001+1) \\
= -(0010) & = -2
\end{align*}
\]
Subtraction in Two's Complement

- A - B = A + (-B) = A + \overline{B} + 1

- 0010 - 0110 = 0010 + (-0110) = 0010 + \overline{0110} + 1
  = 0010 + 1001
  \text{flip} \quad 1100 \quad -4

- 1011 - 1001 = 1011 + 0110
  \text{flip} \quad 0110
  \text{add one} \quad 1100 \quad -2

- 1011 - 0001 = 1011 + 1110
  \text{flip} \quad 0110
  \text{add one} \quad 1010