Damping and Synchronizing Torques

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Electric Torques

- Mechanical torque from turbine is converted into electrical torque
- The electric torque has two components:
  - Synchronizing torque (in phase with power angle)
  - Damping torque (in phase with speed)
- The lack of either torque render the system unstable
- Power system controllers (stabilizers) are used to enhance these torques

Synchronous Machine Model

Mechanical Loop

- Mechanical loop
- Electrical loop

Mechanical Torque

- \( \Delta T_m \)
- \( K_1 \)
- \( \Delta \delta \)
- \( \Delta \omega \)
- \( \Delta v \)
- \( \Delta \theta_f \)
- \( \Delta \theta_d \)
- \( \frac{1}{M_5 + D} \)
- \( \frac{377}{S} \)

Electrical Torque

- \( \Delta \omega \)
- \( \Delta T_e \)
- \( \Delta T_s \)
- \( \Delta \delta \)
Mechanical Loop

\[ \Delta T_m = \Delta T_d + \Delta T_s \]
\[ \Delta T_s = D \Delta \omega + K_1 \Delta \delta \]

- Damping Torque
- Synchronizing Torque

\[ \Delta T_d = 1 \, \frac{S}{M} \]

\[ \Delta \omega \]

\[ \Delta \delta \]

\[ \frac{\Delta \delta}{\Delta T_m} = \frac{377}{M} \left( \frac{S}{S^2 + \frac{D}{M} S + \frac{377 K_1}{M}} \right) \]

\[ \Delta \omega = \frac{377 K_1}{M} \]  
Natural frequency

\[ \xi = \frac{D}{2M \omega_n} \]  
Damping Coefficient

Characteristic Equation

\[ \frac{\Delta \delta(S)}{\Delta T_m(S)} = \frac{C}{S^2 + 2\xi \omega_n S + \omega_n^2} \]

where

\[ \lambda = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2} \]

\( \lambda \) are the roots of the characteristic equation.
Mechanical Loop

\[ \lambda = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2} \]
\[ \lambda = \sigma \pm j \omega_d \]

Where

- \( \sigma \) is the damping
- \( \omega_d \) is the damped frequency

Roots

\[ \lambda = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2} \]
\[ \lambda = \sigma \pm j \omega_d \]
\[ |\lambda| = \omega_n \]

Mechanical Loop

\[ \frac{\Delta T_m}{\Delta \omega} = \frac{K}{1 + MS + D} \]

Assume \( \Delta T_m \) is a step input

\[ \Delta \delta(S) = \frac{C}{S^2 + 2 \xi \omega_n S + \omega_n^2} \]

The time domain solution of the equation is

\[ \Delta \delta(t) = \frac{KC}{\omega_n \sigma} \sin(\omega_d t + \theta) e^{-\xi \omega_d t} \]

\[ \theta = \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right) \]
**Time Response of Mechanical Loop**

\[ \Delta \delta \]

\[ \frac{K_C}{\sigma_i^2} \]

\[ \xi_2 < \xi_i \]

\[ \xi_i \]

\[ \text{Time} \]

**Lack of Damping Torque**

\[ \Delta \delta \]

\[ \frac{K_C}{\sigma_i^2} \]

\[ \xi = 0 \]

\[ \text{Time} \]

**Lack of Damping Torque**

\[ \Delta \delta \]

\[ \frac{K_C}{\sigma_i^2} \]

\[ \xi < 0 \]

\[ \text{Time} \]

**Lack of Synchronizing Torque**

When \( \sigma_n \) is very small, or \( K_i \) is very small:

\[ \sigma_u = \sqrt{\frac{377 K_i}{M}} \]

\[ \lim_{\sigma_n \to 0} \lim_{\omega_n \to 0} \left( \frac{K_C}{\sigma_n^2} - \frac{K_C}{\sigma_n \omega_d} \sin(\omega_d t) e^{-\frac{s}{\Gamma}} \right) = \infty \]

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Lack of Synchronizing Torque

\[ \Delta \delta = \frac{KC}{\sigma_i} \]

Synchronous Machine Model

\[ \Delta T_m + \Delta \omega = \Delta \delta \]

Electrical Loop without VR

\[ \Delta T_a \]

Electrical loop
Electrical Loop with VR

\[ \Delta T_e = \frac{K_e K_f K_d}{1 + \sigma T_d K_f} \Delta \delta \]

\[ \Delta T_e (S) = -K_e K_f K_d \]

\[ \Delta T_e (j \omega) = -K_e K_f K_d \]

\[ G(j \omega) = \frac{K_e K_f K_d}{\sqrt{1 + (\sigma T_d K_f)^2}} \]

\[ \phi(j \omega) = 180^\circ - \tan^{-1}(\sigma T_d K_f) \]

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( G )</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( K_e K_f K_d )</td>
<td>180(^\circ)</td>
</tr>
<tr>
<td>Corner frequency ( 1 )</td>
<td>( \frac{K_e K_f K_d}{\sqrt{2}} )</td>
<td>180(^\circ)-45(^\circ)</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0</td>
<td>90(^\circ)</td>
</tr>
</tbody>
</table>

Damping torque is positive
Synchronizing torque is negative

For stable system
Total damping torque \( T_d \) \( \text{total} = T_d \text{mechanical} + T_d \text{electrical} > 0 \)
Total synchronizing torque \( T_s \) \( \text{total} = T_s \text{mechanical} + T_s \text{electrical} > 0 \)
At corner frequency
\[
\Delta \omega = \Delta \omega_a \Delta T_d = \Delta \omega_a \Delta \delta
\]

Total damping torque \( T_d \) total \( = D + \frac{K_2 K_4}{2} \) > 0
Total synchronizing torque \( T_s \) total \( = K_1 - \frac{K_2 K_4}{2} \) > 0

For stable system

Example
\[
K_1 = 0.54 \quad K_2 = 1.2 \quad K_3 = 0.66 \quad K_4 = 0.7 \quad D = 0
\]

\[
T_d \text{ total} = D + \frac{K_2 K_3 K_4}{2} = 0 + 0.277 > 0
\]

\[
T_s \text{ total} = K_1 - \frac{K_2 K_3 K_4}{2} = 0.54 - 0.277 > 0
\]

The system is stable.

With VR, we can ignore \( K_4 \Delta \delta \)

\[
\Delta T_e \left( S \right) = \frac{-K_1}{\tau_{d_a} \tau_d} \left( \frac{K_2 K_4}{\tau_{d_a} \tau_d} \right) \frac{1 + \tau_{d_a} K_3}{\tau_{d_a} \tau_d K_3} S + \frac{1 + K_1 K_2 K_3}{\tau_{d_a} \tau_d K_3}
\]

\[
\frac{\Delta T_e}{\Delta \delta} = S^2 + \frac{2 \omega_0^2}{\tau_{d_a} \tau_d} S + \omega_0^2
\]
When $K_5 > 0$
Synchronizing Torque is negative up to $m = m_n$

When $K_5 < 0$
Damping Torque is negative up to $m = m_n$

VR could enhance one of the torques but could reduce the other.
Damping and Synchronizing Torques in Multi-Machine System

Electric Torques on i\(^{th}\) Machine

\[ \Delta T_{ei}(s) = \sum_{j=1}^{n} (K_{ij} \Delta \delta_j + K_{ij} \Delta \epsilon_j') \]

- The angle between machine buses determines the magnitude of the synchronizing and/or damping torques